Agent Teams and Evolutionary Computation: Optimizing Semi-Parametric Spatial Autoregressive Models

Tamás Krisztin, Matthias Koch

***Abstract*— Classical spatial autoregressive models share the same weakness as the classical linear regression models, namely it is not possible to estimate non-linear relationships between the dependent and independent variables. In the case of classical linear regression a semi-parametric approach can be used to address this issue. Therefore an advanced semi-parametric modelling approach for spatial autoregressive models is introduced. Advanced semi-parametric modelling requires determining the best configuration of independent variable vectors, number of spline-knots and their positions. To solve this combinatorial optimization problem an asynchronous multi-agent system based on genetic-algorithms is utilized. Three teams of agents work each on a subset of the problem and cooperate through sharing their most optimal solutions. Through this system more complex relationships between the dependent and independent variables can be derived. These could be better suited for the possibly non-linear real-world problems faced by applied spatial econometricians.**

I. Introduction

SPATIAL autoregressive (SAR) models are widely used for empirical problems, caused by spatial autocorrelation. The key element of these techniques is to incorporate a spatial lag into the regression model. Both the classical regression and the SAR model assume that the impact of the independent variables on the depended variable can be modelled in a linear fashion. This might not be true for real world data generating processes, which easily could be non-linear in fashion. Consider for example the housing market: Houses next to each other have autocorrelated prices, since they can be seen as substitutes. In a linear housing price model one would assume that, leaving aside other influences, the marginal ceteris paribus effect of an increase in the net dwelling area on one house price is constant. If for example fixed costs are associated with housing transactions and considered to be significant for housing prices, a linear model is no longer valid.

Since non-linearity is assumed to be a factor in real-world data generating processes, semi-parametric models were introduced into the linear regression framework to address this issue. These semi-parametric regression models are able to cope with most kind of nonlinearity (see for example [2]). Therefore the aim of this paper is to extend the SAR models with semi-parametric modelling techniques. This results in an optimization problem, which, as is argued through the paper, might be suitable to be handled by a team of asynchronous agents, using genetic algorithms.

The suggested semi-parametric spatial autoregressive (SPSAR) estimation-method is based on so called truncated-splines [3] and Akaike Information Criteria (AIC) minimization. The truncated spline and the spatial autoregressive estimators will be calculated via a maximum likelihood (ML). In order to model complex nonlinear relationships between the dependent and independent variables a suitable combinations of the independent variables is chosen and then uses as an argument for the truncated splines. The truncated spline has an optimal selection and an optimized position of knots. Therefore, a combinatorial optimization algorithm is needed.

The SPSAR optimization problem consists of two inter-linked combinatorial tasks: finding the optimal selection and the optimized knots for these splines. If only the first part of this problem is considered in isolation, one would use the Markov-Monte Carlo chain (MCMC) method, like in model selection. Due to the nature of the optimization problem the MCMC approach cannot be used, since it relies on model similarity, which is not present due to the fact that the optimization problem is inter-linked. However an MCMC approach can be viewed as a very basic genetic algorithm. Hence it seems straightforward to use asynchronous agents with genetic algorithms, which can not only handle the non-similarity of the models but also the inter-linked nature of the optimization problem.

The first section details the nonlinear spatial autoregressive models, the SPSAR estimation method and the nature of the optimization problem. In the second section the paper introduces asynchronous multi-agent systems and discusses their characteristics. The third section outlines the asynchronous agent architecture, while the fourth section examines the precursory results of the SPSAR optimization approach.

II. Non-Linear and Semi-Parametric Spatial Autoregressive Models

This section introduces nonlinear spatial econometric models and then suggests semi-parametric modelling to account for the nonlinearity. Since this paper focuses primarily on the asynchronous multi Agent systems, this

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T. Krisztin is with the Vienna University of Business and Economics, Institute for Economic Geography and GIScience, Nordbergstrasse 15, 1090, Vienna, Austria (Tel.: +43-1-313 36-4846), e-mail: tamas.krisztin@wu.ac.at).

M. Koch is with the Vienna University of Business and Economics, Institute for Economic Geography and GIScience, Nordbergstrasse 15, 1090, Vienna, Austria (Tel.: +43-1-313 36-4202), e-mail: matthias.koch@wu.ac.at).

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section should only be seen as a sketch for the actual SPSAR-econometric problem.

Consider the following nonlinear spatial autoregressive model (1):

$$\mathbf{Y}_n = \rho \mathbf{W}_n \mathbf{Y}_n + f(\mathbf{X}_{1,n}, \dots, \mathbf{X}_{k,n}) + \boldsymbol{\varepsilon}_n \quad \text{where } \boldsymbol{\varepsilon}_n \sim i.N(0, \boldsymbol{I}_n \sigma^2) \tag{1}$$

In (1) $\mathbf{Y}_n$ is a n by 1 vector containing the dependent variable. $\mathbf{X}_n$ is a n by k matrix of observations on k independent variables, $\mathbf{W}_n$ is a n by n spatial weighting matrix of known constants, ρ is the spatial autoregressive parameter and $\boldsymbol{\varepsilon}_n$ is an independently normal distributed random vector with zero mean and σ^2 variance. $f(\mathbf{X}_{1,n}, \dots, \mathbf{X}_{k,n})$ is a nonlinear continuous function with continuous derivatives form $\mathbb{R}^{n\times k} \to \mathbb{R}^{n\times 1}$. Additionally it is assumed that $\mathbf{X}_n$ only contains metric variables. For notational simplicity the constant term of the spatial regression model is ignored in this section.

Assume that $\mathbf{W}_n$ is either row or maximum row standardized and that the true parameter of ρ is smaller one in absolute value. Therefore, (1) can be solved for $\mathbf{Y}_n$ and this results in (2)

$$\mathbf{Y}_n = (\mathbf{I}_n - \rho \mathbf{W}_n)^{-1} f(\mathbf{X}_{1,n}, \dots, \mathbf{X}_{k,n}) + (\mathbf{I}_n - \rho \mathbf{W}_n)^{-1} \boldsymbol{\varepsilon}_n \tag{2}$$

Since the specific form $f(\mathbf{X}_{1,n}, \dots, \mathbf{X}_{k,n})$ is not known a finite truncated Taylorseries is used. Since this series is not practicable, a series of truncated splines $g_i^{\overline{k_\iota}}(\overline{\mathbf{X}}_i, \gamma)$ of optimized length m, where $\overline{k_\iota}$ is the set containing the optimized knots for the truncated spline and $\overline{\mathbf{X}}_i \in \{\mathbf{X}_{\mathrm{j,n}}^{\mathrm{l}} \odot \mathbf{X}_{\mathrm{o,n}}^{\mathrm{h}} \mid (\mathrm{j,o}) \in \mathrm{T_k} \times \mathrm{T_k}, (\mathrm{l,h})_{\mathrm{j}\neq\mathrm{o}} \in \mathrm{T_3} \times \mathrm{T_3}\} \cup \{\mathbf{X}_{\mathrm{j,n}}^{1}, \dots, \mathbf{X}_{\mathrm{j,n}}^{3} \mid \mathrm{j} \in \mathrm{T_k}\}$ where $\mathrm{T_x} = \{1,2,\dots,\mathrm{x}\}$ is used. Hence $f(\mathbf{X}_{1,n}, \dots, \mathbf{X}_{k,n})$ will be approximated by $\sum_{i=1}^{m} g_i^{\overline{k_\iota}}(\overline{\mathbf{X}}_i, \gamma)$. Since $g_i^{\overline{k_\iota}}(\overline{\mathbf{X}}_i, \gamma)$ represents a truncated spline, $\sum_{i=1}^{m} g_i^{\overline{k_\iota}}(\overline{\mathbf{X}}_i, \gamma)$ must have a linear representation: $\sum_{i=1}^{m} g_i^{\overline{k_\iota}}(\overline{\mathbf{X}}_i, \gamma) = \mathbf{Z}_{\mathrm{n}} \bar{\gamma}$ for given vectors $\overline{\mathbf{X}}_i$, the set $\overline{k_\iota}$ and the length m. If this approximation of $f(\mathbf{X}_{1,n}, \dots, \mathbf{X}_{k,n})$ is used (2) can be rewritten to (3)

$$\mathbf{Y}_n \approx \rho \mathbf{W}_n \mathbf{Y}_n + \mathbf{Z}_{\mathrm{n}} \bar{\gamma} + \boldsymbol{\varepsilon}_n \tag{3}$$

Estimators for ρ, $\bar{\gamma}$ and σ^2 (estimators are denoted with ^) in (3) can be found via ML. ML leads to the following maximization problem (4) (Le Sage and Pace, 2009):

$$\begin{pmatrix} \hat{\rho} \\ \hat{\gamma} \\ \hat{\sigma}^2 \end{pmatrix} = \max_{\rho, \bar{\gamma}, \sigma^2} \left\{ \frac{1}{(2\pi)^{\frac{n}{2}} \det(\mathbf{S}(\rho)^{-1}\sigma)} \exp\left(-\frac{1}{2\sigma^2} (\mathbf{S}(\rho)\mathbf{Y}_n - \mathbf{Z}_{\mathrm{n}}\bar{\gamma})' (\mathbf{S}(\rho)\mathbf{Y}_n - \mathbf{Z}_{\mathrm{n}}\bar{\gamma}) \right) \right\} \tag{4}$$

where $\mathbf{S}(\rho) = (\mathbf{I}_n - \rho \mathbf{W}_n)$. With the estimators $\hat{\rho}$, $\hat{\gamma}$ and $\hat{\sigma}^2$ the AIC can be calculated. $\mathbf{Z}_{\mathrm{n}}\hat{\gamma}$ is considered a good estimator for $f(\mathbf{X}_{1,n}, \dots, \mathbf{X}_{k,n})$ if a minimal AIC is found. Since most of the econometric issues like ML are already sufficiently solved, section four discusses the optimization procedure for finding optimal $\overline{\mathbf{X}}_\iota$ and number and position of the truncated spline knots.

III. ASYNCHRONOUS MULTI-AGENT SYTEMS

This section provides a brief introduction to agents, multi-agent systems (MAS) and a more specific overview of asynchronous MAS for solving large combinatorial optimization problems.

The definition of agents is laid down by [7], namely an agent is defined by possessing one or more of the four characteristics:

- Autonomy is the agent's ability to work without human interaction and have a control of their own state and actions.
- Social ability is the ability to communicate with other agents.
- Reactivity denotes the ability to respond to actions and to perceive the environment.
- Pro-Activeness is the agent's ability to work towards a goal and take initiative in actions.

A multi-agent system is a collection of loosely coupled agents, who cooperate to solve a problem [5]. In MAS each agent has only limited information and problem solving capacity, so that the posed problem can only be solved through cooperation. Furthermore there is no central entity that manages the system, instead data and problem-solving are decentralized and managed by individual agents.

Asynchronous problem-solving teams (ATeams), have been proposed by [6]. They are a form of MAS, where the system-wide current best solutions of a problem are stored in a central memory. Problem-solving agents try to generate more optimal solutions, each agent with another problem-solving algorithm, while destroyer agents are deleting sub-optimal solutions from the central memory. The architecture of such an ATeam is asynchronous, agents act in an autonomous way and they exchange information through the shared memory.

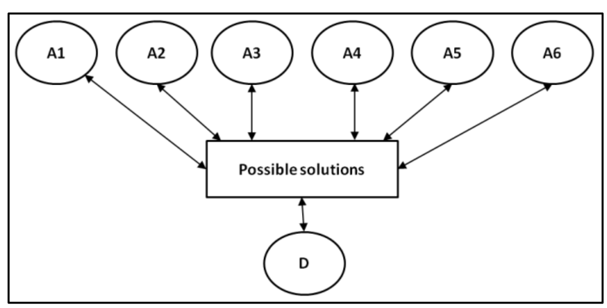

Fig. 1. Architecture of an ATeam

Fig 1 illustrates this principle, where agents A1 through A6 are working to solve the problem using algorithms a1 through a6. They add their most optimal solutions to the solution population in the memory M. Other agents review these solutions periodically and try to come up with more optimal solutions. The destroyer agent D is checking the population of solutions and deletes any inferior solution which is below a threshold t.

For more complicated problems, multiple ATeams can be employed, with each team of agents working on a subset of a problem. Communication between the teams depends on the organization of the problem and by what degree the subsets of the problem depend on each other. Fig. 2 shows two agent teams, the first team is A1 through A3 and the second team A4 through A6, cooperating in this way. The second team of agents builds on the population of the first and an coordinator agent C provides subproblems for the second team from the solutions of the first team.

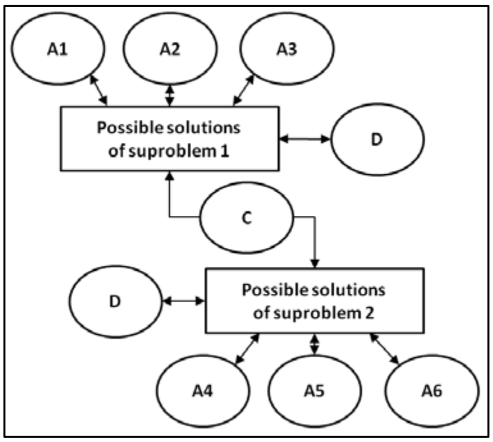

Fig. 2. Multiple ATeams cooperating on subsets of a problem

In some sense ATeams are similar to blackboard systems, where problem-solvers cooperate by posting the results of their calculations on a blackboard. In ATeams though the agents operate independently and unlike in blackboard systems, there is no central instance of control and agents work independently from each other (Aydin et al., 2004).

It is possible to combine ATeams with evolutionary methods, such as genetic algorithms. This can be done either with each agent as an instance of an algorithm or each agent performing the individual steps of the algorithm. In such a case one agent would implement the population selection, while others implement crossovers and genetic mutations. It is easy to implement hybrid methods, with different evolutionary algorithms, in an ATeam, since agents can be easily added or substracted from the system; this offers a flexible way of solving complex optimization problems through addition of different selection and crossover methods [1].

Talukdar et al. [6] used the well-known shortest path problem as a benchmark for ATeams. By applying this methodology to the problem they demonstrated that the solutions provided by ATeams offer more reachability and can solve the problem in a more efficient manner, than more conventional methods. Other applications of similarly structured asynchronous MAS can be found in the supply-chain literature [4,1] where they are still popular. We are not aware of any other applications of ATeams in the field applied spatial econometrics.

IV. SOLUTION METHODOLOGY

To optimize the problem of semi-parametric spatial autoregressive models, which we introduced in the previous section, we propose two types of asynchronous agents, each working on a subset of the optimization problem. The first type of agent attempts to optimize the selection of splines, while the second type of agent tries to find the optimal variable vectors for the selected number of splines and positions. Based on the AIC of these results, each team of agents attempts to improve upon the solution. Each cyle the worst performing splines are deleted from the population.

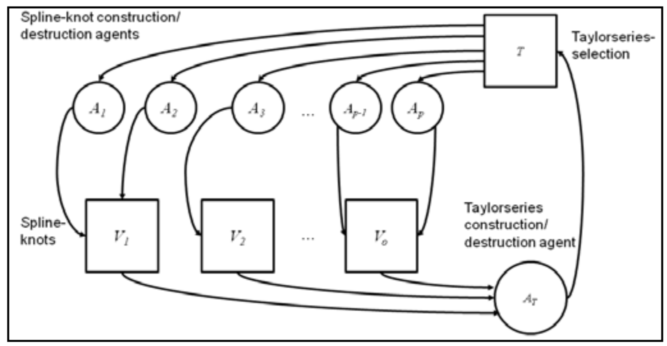

Fig. 3. Architecture of the system

The system starts with a pool of randomly selected population samples, with uniform distribution. For an overview of the system architecture, see Fig. 3. Each spline in the problem has an own population set. One or multiple agents are associated with each spline population. These agents attempt to improve the population by the following algorithm:

1. Set $t=0$
2. Initialize spline-knot population V
3. Evaluate V using random Taylorseries-selections
4. While (best AIC improvement$<g_{AIC}$)

4.1 Selection

4.1.1 Select V^1 random spline-knots from V (t) for tournament

4.1.2 Select T^1 random Taylorseries-selection from T

4.1.3 Evaluate AIC of V^1 using Taylorseries-selections T^1

4.2 Select best spline-knots,V^2, and worst spline-knots, V^3, from V^1 for recombination

4.3 Recombine spline-knots in V^2, thus form V^4

4.4 Mutate V^4, forming V^5

4.5 Modify Population

4.5.1 Set $V(t+1)=V$

4.5.2 Insert V^5 into $V(t+1)$

4.5.2 Remove V^3 from $V(t+1)$

4.6 Set $t=t+1$

The Taylorseries selection agent uses the best AICs from each population and attempts to optimite each cycle.

The concept of ATeams is implemented by employing different types of spline-knot construction/destruction agents. There are three types of such agents in the system; each of them uses different crossover methods:

- The first type of agent (g1) uses single-point crossover for creating new solutions,
- the second agent type (g2) employs two-point crossover and
- the third type of agents (g3) uses a random crossover method, whereby a binary random vector - corresponding to the length of the first parent - is created. Where the vector has a value of 1, the matching value of the first parent is chosen, else the equivalent value of the second parent is selected for the offspring.

V. RESULTS

So far the following process has been considered, as a test-case for the SPSSR method:

$$y_i \approx \sum_{j=1}^{n} .4 w_{ij} y_j + \sin\left(\frac{x_1 x_2}{10}\right) + \varepsilon_i \qquad (5)$$

where x1 and x2 are uniformly distributed between 0 and 1, n=100, $\mathbf{W}_n$ represents a spatial one forward one behind pattern and a noise level σ of 0.05. If (5) is simulated figure (Fig. 4) can be seen as representative and shows the difference between a SAR and SPSAR in-sample forecast.

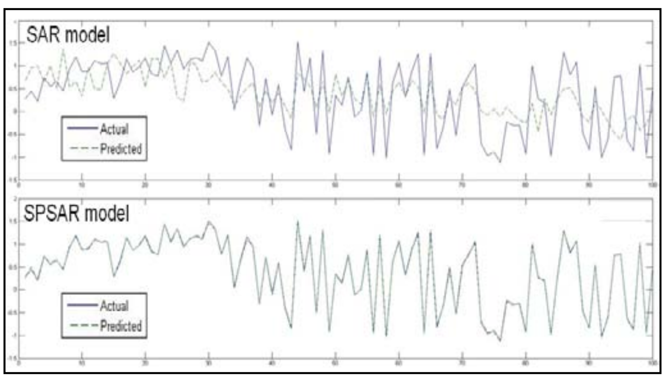

Fig. 4. SAR and SPSAR model forecasts

The SPSAR model gives a far better visual fit of the data than the SAR model. The AIC values of the best performing agent after 50 cycles are illustrated in Fig. 4. The trend in this figure clearly shows a fast convergence at the beginning cycles and then a gradual improvement of the AIC scores.

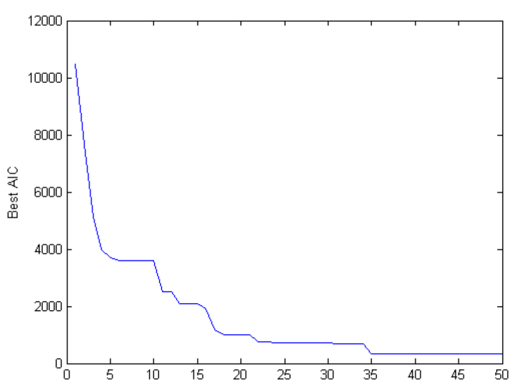

Fig. 5. AIC improvement of the best performing agent after 50 cycles

The same trend can be observed, when looking at the overall performance of thirty agents in the system (Fig. 6).

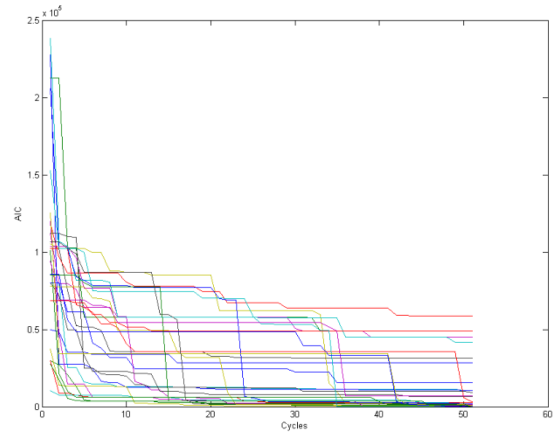

Fig. 6. Overall AIC improvement of thirty agents

VI. CONCLUSIONS

This paper derives an optimization for semi-parametric spatial autoregressive models, through asynchronous multi-agent teams. The agent teams employ genetic algorithms and cooperate to find the optimal solution for this large combinatorial optimization problem.

This agent-based model offers an elegant method for applied spatial econometrics. Through combined agent teams the problem can be subdivided and solved on separate levels. In addition it is also possible to try other then evolutionary methods for the agents, even combining hybrid approaches. Due to the characteristics of ATeams such an extension can be implemented to utilise the proposed methodology for other spatial econometric problems.

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